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Differential evolutive problems in spaces with singularities

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List of Abstracts

Abstracts

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Continuos primitives for differential forms in Euclidean spaces and Heisenberg groups.

In this talk I will present a group of results obtained in collaboration with Bruno Franchi (Bologna) and Pierre Pansu (Orsay) that are the analytical version of the well known topological problem whether a given closed form is exact: whether a primitive can be upgraded to one which satisfies estimates. Indeed, in the last few years, in collaboration with Bruno Franchi and Pierre Pansu, we have proved some global and interior (p, q)- Poincaré and Sobolev inequalities for differentialforms in the so-called Rumin complex in Heisenberg groups. The first bunch of results have been obtained for $1 \leq p < Q$ (or $p < Q/2$, depending on the degree of the forms) where Q is the homogeneous group dimension of the group. Also the limiting case $q = \infty$ has been considered. The endpoint situation $q = \infty$ corresponds to the exponent $p = Q$ (or $p = Q/2$). Notice that already for functions in the Euclidean setting, when $p = N$ a Poincaré inequality fails to hold (Trudinger inequality), but Burgain & Brezis proved that Poincaré inequality on the N-torus still holds for vector fields when $p = N$ and $q = \infty$. Even more, the primitive can be taken continuous. The global version on $\mathbb{R}^N$ itself is even slightly stronger: the primitive can be taken to be continuous and to tend to zero at infinity, as proved by Moonens and Picon. We show that the result of Moonens and Picon in $\mathbb{R}^N$ holds for closed differential forms of any degree. In addition, we prove analogous results for the Rumin's complex in Heisenberg groups. In particular, that it possible to upgrade bounded primitives to bounded and continuous primitives vanishing at infinity.

Ugo Boscain
Sorbonne Université, Paris VI

Heat and Schroedinger evolution on surfaces embedded in 3D contact SR manifolds.

In this talk I consider a surface embedded in a 3D contact sub-Riemannian manifold. Such a surface inherits a field of direction (with norm) from the ambient space.

This field of directions is singular at characteristic points (i.e., where the surface is tangent to the set of admissible directions). In this talk we will study when the normed field of directions permits to give to the surface the structure of metric space (of "SNCF" type). I will also study how to define the heat and the Schroedinger equation on such a structure and if the singular points are "accessible" or not. When the singular points are accessible we will study self-adjoint extensions with Kirchhoff like boundary conditions.

Roberta Ghezzi
Università degli Studi di Roma "Tor Vergata"

Regularity theory and geometry of unbalanced optimal transport.

Using the dual formulation only, we show that the regularity of unbalanced optimal transport also called entropy-transport inherits from the regularity of standard optimal transport. We provide detailed examples of Riemannian manifolds and costs for which unbalanced optimal transport is regular. Among all entropy-transport formulations, Wasserstein-Fisher-Rao (WFR) metric, also called Hellinger-Kantorovich, stands out since it admits a dynamic formulation, which extends the Benamou-Brenier formulation of optimal transport. After demonstrating the equivalence between dynamic and static formulations on a closed Riemannian manifold, we prove a polar factorization theorem, similar to the one due to Brenier and Mc-Cann. As a byproduct, we formulate the Monge-Ampere equation associated with WFR metric, which also holds for more general costs. Last, we study the link between c-convex functions for the cost induced by the WFR metric and the cost on the cone. The main result is that the weak Ma-Trudinger-Wang condition on the cone implies the same condition on the manifold for the cost induced by WFR.

Joint work with T. Gallouët (INRIA Saclay) and F.-X. Vialard (Université Gustave Eiffel Paris)

Enrico Le Donne
Université de Fribourg

Variational Curve Flows in Carnot Groups.

Starting with some curve in a Carnot group, we aim to flow the curve to shorten it while keeping its endpoints fixed. We consider Carnot groups equipped with sub-Riemannian distances, and seek to shorten both length and energy. Geodesics will be stationary points of the flow. As expected, abnormal curves will be obstructions to the theory, and in fact, they will cause the PDE system defining the flow to be discontinuous. However, I will discuss the system's equation, few properties, and its short-time existence.

These results are in collaboration with L. Capogna (Smith).

Giuseppe Longo
Ecole Normale Supérieure, Paris

The physical singularity of life.

In physics, many notions of "emergence" have been proposed: from Turing and Thom's analysis of morphogenesis to self-organization in thermodynamics, to various dynamics of "networks". The singularity and specificity of the living state of matter will be emphasized, as well as the difference between "emergence" in physics and "production of novelty" in biology. In Darwinian evolution, in particular, the conceptual (and, possibly, mathematical) framework requires the construction of new perspectives, due to the nature of the relevant observables and their historical specificity. The question of "unification" with some of the numerous theories of the inert involved in biological intelligibility will be touched upon, building bridges and proposing conceptual dualities.

Lorenzo Mazzieri
Università di Trento

Geometric inequalities and potential theory.

We describe through some selected examples an approach based on potential theory toward the proof of relevant geometric inequalities, holding in classical and curved frameworks. Time permitting, we also discuss some applications of interest in general relativity, including the positive mass theorem and the Riemannian Penrose inequality.

Andrea Mondino
University of Oxford

On the unification of Riemannian, Finslerian and Sub-Riemannian synthetic Ricci lower bounds.

Motivated by progress on sub-Riemannian geometry on the one hand, and on synthetic Ricci curvature lower bounds on metric measure spaces via optimal transport on the other hand, Villani (Bourbaki Seminar 2017) proposed as open problem to obtain a "Great unification" of synthetic Riemannian, Finslerian and Sub-Riemannian synthetic Ricci lower bounds. The goal of the talk is to report on joint work with Barilari (Padova) and Rizzi (SISSA) on the topic.

Roberto Monti
Università di Padova

Higher order Goh conditions for singular extremals of corank 1.

We present a joint work with Francesco Boarotto and Alessandro Socionovo (ARMA 2024) on Goh conditions of order $m \geq 3$ for strictly singular length-minimizing curves of corank 1. The results rely upon new open mapping theorems for maps with regular mth differential.

Ludovic Rifford
Université Côte d'Azur

On the minimizing Sard Conjecture in sub-Riemannian geometry.

After recalling the notions of minimizing geodesics and singular horizontal curves in sub-Riemannian geometry, we will discuss various versions of the so-called Sard conjecture and present several result dealing with the minimizing Sard Conjecture. The proof of our main result will be sketched, it relies on tools from non-smooth analysis and geometric measure theory.

Luca Rizzi
SISSA, Trieste

Sard properties for polynomial maps in infinite dimension with applications to the Sard conjecture in Carnot groups.

Sard's theorem asserts that the set of critical values of a smooth map from one Euclidean space to another one has measure zero. It is well-known, however, that when the domain is infinite dimensional and the range is finite dimensional the property may fail. I will present a recent work with A. Lerario and D. Tiberio (SISSA), establishing sharp quantitative criteria for the validity of Sard's theorem in this setting. Our motivation comes from sub-Riemannian geometry and we provide in particular applications of our results to the Sard conjecture on Carnot groups.

Alessandro Sarti
CAMS, EHESS, Paris

Differential heterogenesis.

The transition from physical and structural dynamics to heterogenetic dynamics will be discussed. Unlike physical dynamics, where the space of possibility is fixed a

priori, and structural dynamics, where it becomes a space of control, in heterogenesis the space of possibility changes in space and time. It deals with the axis of the historicity of dynamics. This is the axis of phylogenesis in the evolution of species, of cortical plasticity in neuroscience, of the invention of concepts in cognitive science. Examples of models from cognitive neuroscience will be proposed.

Joint work with G.Citti and D.Piotrowski.