Partial differential equations and related functional inequalities
Titles and abstracts of talks

Angela Alberico

Title: Fractional Orlicz-Sobolev Embeddings

Abstract: The optimal Orlicz target space and the optimal rearrangement-invariant target space are exhibited for embeddings of fractional-order Orlicz-Sobolev spaces. Both the subcritical and the supercritical regimes are considered. In particular, in the latter case the relevant Orlicz-Sobolev spaces are shown to be embedded into the space of bounded continuous functions in $\mathbb{R}^n$. This is a joint work with Andrea Cianchi, Lubos Pick and Lenka Slavikova.

Denis Bonheure

Title: Classification of radial blow-up at the first critical exponent for the Lin-Ni-Takagi equation in a ball.

Abstract: We investigate the behaviour of radial positive solutions to the Lin-Ni-Takagi problem in the ball $B_R \subset \mathbb{R}^N$:

$$\begin{cases} -\triangle u_p + u_p = |u_p|^{p-2}u_p & \text{in } B_R \\ \partial_\nu u_p = 0 & \text{in } \partial B_R, \end{cases}$$

when $p \to 2^* = \frac{2N}{N-2}$. We obtain a complete classification of finite energy blowing-up solutions to this problem. In particular, we show that if $p \geq 2^*$, then solutions are compact provided that $N \geq 7$. We also give an interpretation of our results in view of the bifurcation analysis of Bonheure, Grumiau and Troestler in Nonlinear Anal. 147 (2016). Joint work with Jean-Baptiste Castéras (Lisbon), Bruno Premoselli (ULB)

Iwona Chlebicka

Title: Absence of Lavrentiev's phenomenon in anisotropic spaces

Abstract: I will discuss a new result on approximation implying the absence of Lavrentiev's phenomenon for a natural range of anisotropic functionals. In fact, we provide a universal and precise description of a family of inhomogeneous and fully anisotropic weak N-functions that generate function spaces of Musielak-Orlicz-Sobolev type in which smooth functions are dense in a natural topology.

Giulio Ciraolo

Title: Classification results for critical p-Laplace equations

Abstract: We consider critical p-Laplace type equations arising from Sobolev type inequalities. Nonnegative solutions of these equations are unique up to scaling and translation and this property can also be interpreted as a rigidity result. Classification results are well-known and have been obtained by using the method of moving planes. In this talk, we will discuss two approaches which do not make use of the method of moving planes and allow us to give a complete classification of the solutions in an anisotropic setting

as well as to a suitable generalization of the problem in convex cones. An application to Caffarelli-Kohn-Nirenberg (CKN) inequalities will be also discussed.

Cristiana De Filippis

Title: Schauder estimates for any taste

Abstract: So-called Schauder estimates are a standard tool in the analysis of linear elliptic and parabolic PDE. They have been originally obtained by Hopf (1929, interior case), and by Schauder and Caccioppoli (1934, global estimates). The nonlinear case is a more recent achievement from the '80s (Giaquinta & Giusti, Ivert, Lieberman, Manfredi). All these classical results hold in the uniformly elliptic framework. I will present the solution to the longstanding open problem (70s) of proving estimates of such kind in the nonuniformly elliptic setting. I will also cover the case of nondifferentiable functionals and of mixed local and nonlocal problems. From joint work with Giuseppe Mingione (University of Parma).

Jean Dolbeault

Title: Stability estimates in critical functional inequalities

Abstract: Optimal functions and optimal constants are known in various functional inequalities, for instance in Sobolev's inequality. In 1991, Bianchi and Egnell proved that the deficit in Sobolev's inequality (with $p = 2$, on the Euclidean space), that is, the difference of the two sides of the inequality written with the optimal constant, controls the distance to the manifold of the Aubin-Talenti optimal functions in a strong Sobolev norm. An issue with the method is that the new constant is so far unknown. The purpose of this lecture is to review some examples of related functional inequalities in which one can at least give an estimate of the stability constant, with a special emphasis on critical inequalities. In the case of Sobolevs inequality, a result obtained in collaboration with M.J. Esteban, M. Loss and R. Frank will be presented.

Tobias König

Title: Multibubble blow-up analysis for the Brezis–Nirenberg problem in three dimensions

Abstract: For a smooth bounded domain $\Omega \subset \mathbb{R}^3$ and smooth functions a and V, consider a sequence of positive solutions u_ϵ to $-\Delta u_\epsilon + (a + \epsilon V)u_\epsilon = u_\epsilon^5$ on Ω with zero Dirichlet boundary conditions, which blow up as $\epsilon \to 0$. Brezis and Peletier (1989) asked how the asymptotic behavior of such a sequence having exactly one blow-up point can be characterized. In this talk I will present a new result which gives the sharp blow-up rate and characterizes the location of concentration points in the more general case of multiple blow-up points, thereby establishing a complete picture of blow-up phenomena in this framework. This is joint work with Paul Laurain (Universit Paris Cit and ENS Paris).

Enno Lenzmann

Title: On uniqueness for the prescribed Q-curvature problem in one dimension.

Abstract: We consider the nonlocal Liouville equation $\sqrt{-\Delta}w = Ke^w$ on the real line, where K is a given function that can be regarded as a prescribed Q-curvature. For positive and constant K = const. ¿ 0, the solutions are explicitly known and unique up to symmetries. In this talk, we shall present a uniqueness result for a broad class of positive

and non-constant Ks by exploiting a surprising connection to completely integrable PDEs of Calogero-Moser type. This talk is based on joint work with Maria Ahrend (Basel) and Patrick Grard (Orsay).

Petru Mironescu

Title: Sobolev maps to manifolds

Abstract: Sobolev spaces $W^{s,p}$ of maps with values into a compact manifold naturally appear in geometry and material sciences. They exhibit qualitatively different properties from scalar Sobolev spaces: in general, there is no density of smooth maps, and standard trace and lifting theory fail. We will review some of their basic properties, with focus on the case where $s < 1$, in which harmonic analysis tools combined with geometric considerations are quite effective. In particular, we discuss the factorization of unimodular maps, which can be seen as a geometric version of paraproducts.

Xavier Ros Oton

Title: The singular set in the Stefan problem

Abstract: The Stefan problem, dating back to the XIXth century, is probably the most classical and important free boundary problem. The regularity of free boundaries in the Stefan problem was developed in the groundbreaking paper (Caffarelli, Acta Math. 1977). The main result therein establishes that the free boundary is C^∞ in space and time, outside a certain set of singular points. The fine understanding of singularities is of central importance in a number of areas related to nonlinear PDEs and Geometric Analysis. In particular, a major question in such a context is to establish estimates for the size of the singular set. The goal of this talk is to present some new results in this direction for the Stefan problem. This is a joint work with A. Figalli and J. Serra.

Mathias Schäffner

Title: Regularity for nonuniformly elliptic equations and applications in homogenization

Abstract: I discuss local regularity properties for solutions of certain nonuniformly elliptic equations in divergence form. Assuming certain integrability conditions on the coefficient field, we obtain local boundedness and Harnack inequality. The assumed integrability assumptions are essentially optimal and improve upon classical results due to Trudinger from the 1970s. Moreover, I will discuss some applications and generalizations to nonlinear nonuniformly elliptic equations. Furthermore, I will discuss how those results can be used to establish large-scale regularity properties for solutions of uniformly elliptic nonlinear equations with random or periodic coefficients.

Cristina Trombetti

Title: A symmetry result in a free boundary problem

Abstract: We study a shape optimization problem involving a solid $K \subset \mathbb{R}^n$ which has constant temperature and is surrounded by a layer of insulating material which obeys a generalized boundary heat transfer law. We minimize the energy of such configurations among all (K, Ω) with prescribed measure for K and Ω, and without topological or geometrical constraints. In the convection case (corresponding to Robin boundary conditions on $\partial\Omega$) we obtain a full description of minimizers. In the general case, we prove the existence and regularity of solutions and we give a partial description of minimizers.