ACCADEMIA NAZIONALE DEI LINCEI

THE KOREAN ACADEMY OF SCIENCE AND TECHNOLOGY

CONFERENCE

NEW FRONTIERS IN ALGEBRA AND GEOMETRY

15-16 OCTOBER 2025

A B S T R A C T

Scientific Organizing Committee: Enrico ARBARELLO (Linceo, Sapienza Università di Roma), Fabrizio CATANESE (coordinatore, Linceo, Universität Bayreuth), Ciro CILIBERTO (Linceo, Università degli Studi Tor Vergata), Jong Hae KEUM (CMC-Center for Mathematical Challenges – KIAS-Korea Institute for Advanced Study), Emanuele MACRÌ (Linceo, Université Paris-Saclay), Kieran Gregory O'GRADY (Linceo, Sapienza Università di Roma), Claudio PROCESI (Linceo, Sapienza Università di Roma), Claire VOISIN (Lincea, Institut de Mathématiques de Jussieu-Paris), Umberto ZANNIER (Linceo, Scuola Normale Superiore).

PROGRAMME

A primary goal of this Meeting will be to witness and foster the existing close ties and cooperations between Korean and Italian mathematicians, in the fields related to Algebra and Geometry, with a particular emphasis on Algebraic Geometry, a major field of mathematical research and knowledge.
A second goal, in a historical moment where scientific research is extremely specialized, also in mathematics, is to be a forum of frank and open interdisciplinary discussion in the field of mathematics, highlighting the interconnections of algebra, geometry and combinatorics, leading to a unified building of knowledge meant to reverberate on other disciplines, as Physics for instance.
Algebra, geometry and combinatorics are all fields of exact calculations, as opposed to the numerical approximations and iterative methods: and the complexity of these calculations is overcome by the overarching principle of symmetry and geometrical representations.
In the end, algebra may invoke the construction of combinatorial objects, or, as in the theory of linear series, and the theory of crystallographic groups, of convex geometry.
In turn, algebro-geometric methods, as the hard Lefschetz theorems, may lead to the solution of long standing problems in commutative algebra and combinatorics (as witnessed by a recent Fields medal award).
Groups of transformations are the key to the Symmetry principle, and here we encounter finite groups of symmetries, as well as infinite Lie Groups, and their differential geometric properties, and related differential equations.
Complex geometry, but also positive characteristic and arithmetic geometry intertwine to lead to the solution to deep open problems: and the scenario is by and large still far from being fully uncovered.

Wednesday 15 October

10.00 *Welcome addresses*

Carlo DOGLIONI (Vice President of the Accademia Nazionale dei Lincei)

Jeong Han KIM (Vice President of KAST - The Korean Academy of Science and Technology)

10.15 Jun-Muk HWANG (IBS - Institute for Basic Science): *Holomorphic symplectic geometry of elliptic surfaces*

10.45 *Discussion*

10.55 Yongnam LEE (IBS-Center for Complex Geometry): *Compact moduli of elliptic surfaces with multiple fibers*

11.25 *Discussion*

11.35 Coffee break

11.50 Sijong KWAK (KAIST - Korea Advanced Institute of Science and Technology.): *Gonality Sequence of A Smooth Curve and Syzygies of Secant Varieties*

12.20 *Discussion*

12.30 Ciro CILIBERTO (Linceo, Università degli Studi Tor Vergata): *Boundary divisors in the moduli spaces of stable Horikawa surfaces with $K^ 2 = 2p_g - 3$*

12.50 *Discussion*

13.00 Break

14.30 Jong Hae KEUM (CMC-Center for Mathematical Challenges – KIAS-Korea Institute for Advanced Study): *Combinatorially minimal mori dream surfaces of general type*

15.00 *Discussion*

15.10 Young-Hoon KIEM (KIAS - Korea Institute for Advanced Study): *Generalized sheaf counting*

15.40 *Discussion*

15.50 Coffee break

16.05 Jeong Han KIM (KIAS-Korea Institute for Advanced Study - Vice President of KAST-The Korean Academy of Science and Technology): *Asymptotic bounds for Ramsey numbers*

16.35 *Discussion*

16.45 Kieran Gregory O'GRADY (Linceo, Sapienza Università di Roma): *General polarized HK varieties of Type $K3^{[a^2+1]}$ as moduli spaces of sheaves*

16.55 *Discussion*

Thursday 16 October

10.00 Claudio PROCESI (Linceo, Sapienza Università di Roma): *From determinants to Cayley Hamilton algebras*

10.30 *Discussion*

10.40 Fabrizio CATANESE (Linceo, Universität Bayreuth): *Fake homology and homotopy quadrics, Surfaces isogenous to a Product, and numerically trivial automorphisms of surfaces*

11.10 *Discussion*

11.20 Coffee break

11.35 Barbara FANTECHI (Lincea, SISSA Trieste): *Smoothing of projective schemes with lci singularities*

12.05 *Discussion*

12.15 Lucia CAPORASO (Università di Roma Tre): *Hyperbolicity of log-surfaces and hypertangency of curves*

12.45 *Discussion*

12.55 Break

14.30 Paola FREDIANI (Università di Pavia): *Asymptotic directions in the moduli space of curves*

15.00 *Discussion*

15.10 Rita PARDINI (Università di Pisa): *Surfaces with canonical map of odd degree*

15.40 *Discussion*

15.50 Jihun PARK (Pohang University of Science and Technology - POSTECH)): *Unipotent group actions on affine hypersurfaces*

16.20 *Discussion*

16.30 Coffee break

16.45 Jongil PARK (Seoul National University - SNU): *A study of rational homology projective planes*

17.15 *Discussion*

ROMA – PALAZZO CORSINI- VIA DELLA LUNGARA, 10
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All information for attending the conference is available at:
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Per partecipare al convegno è necessaria l'iscrizione online
Fino alle ore 10 è possibile l'accesso anche da Lungotevere della Farnesina, 10
I lavori potranno essere seguiti dal pubblico anche in streaming
Online registration is required to attend the conference.
Until 10 a.m. access is also possible from Lungotevere della Farnesina 10.
The conference can also be followed by streaming.

L'attestato di partecipazione al convegno viene rilasciato esclusivamente a seguito di partecipazione in presenza fisica e deve essere richiesto al personale preposto in anticamera nello stesso giorno di svolgimento del convegno
The certificate of participation in the conference will be issued only following on attendance in presence and must be requested to the staff in charge on the same day of the conference.

Holomorphic symplectic geometry of elliptic surfaces

Jun-Muk HWANG (IBS - Institute for Basic Science)

We say that the formal principle with convergence holds for a compact complex submanifold A in a complex manifold X, if a formal isomorphism between the formal neighborhood of A in X and that of another embedding of A in any complex manifold always converges, namely, the formal isomorphism can be extended to a biholomorphic map between suitable Euclidean neighborhoods. Hirschowitz and Commichau-Grauert showed that the formal principle with convergence holds if the normal bundle of the submanifold is sufficiently positive. We discuss the problem when the normal bundle is only weakly positive, but the submanifold satisfies certain geometric conditions. Our main interest is when the submanifold is a general minimal rational curve in a uniruled projective manifold, such as a general line on a rational homogeneous space or a projective hypersurface of low degree. This is a joint work with Jaehyun Hong.

Compact moduli of elliptic surfaces with multiple fibers

Yongnam LEE (IBS-Center for Complex Geometry)

Motivated by Miranda and Ascher-Bejleri's works on compactification of moduli space of rational elliptic surfaces with a section, we study to construct compact moduli space of elliptic surfaces with multiple fibers.
In a joint work with Donggun Lee, we propose an approach to understanding limit surfaces by Q-Gorenstein smoothing of slc surfaces. And in ongoing work with Dori Bejleri and Donggun Lee, we study compact moduli spaces of the rational elliptic surfaces of index 2 and of Enriques surfaces with bisections using the theory of twisted stable maps.

Gonality Sequence of A Smooth Curve and Syzygies of Secant Varieties

Sijong KWAK (KAIST - Korea Advanced Institute of Science and Technology.)

In this talk, I would like to introduce the gonality sequence of a smooth projective curve and its relation to the syzygies of secant varieties of the curve embedded by a line bundle of sufficiently large degree. More precisely, it can be shown that together with Ein–Niu–Park's theorem, the gonality sequence of a smooth projective curve completely determines the shape of the minimal free resolutions of secant varieties of the curve of sufficiently large degree. This is a natural generalization of the gonality conjecture on syzygies of smooth projective curves established by Ein–Lazarsfeld and Rathmann to the secant varieties. This is a joint work with Junho Choe and Jinhyung Park.

Boundary divisors in the moduli spaces of stable Horikawa surfaces with $K^ 2 = 2p_g - 3$

Ciro CILIBERTO (Linceo, Università degli Studi Tor Vergata)

In this talk I will describe the normal stable surfaces with $K^2=2p_g-3$ whose only non canonical singularity is a cyclic quotient singularity of type $\frac{1}{4k}(1,2k-1)$ and the corresponding locus $\mathfrak D$ inside the KSBA moduli space of stable surfaces. The main result is the following: for $p_g\ge 15$, (1) a general point of any irreducible component of $\mathfrak D$ corresponds to a surface with a singularity of type $\frac 14(1,1)$, (2) the closure of ${\mathfrak D}$ is a divisor contained in the closure of the Gieseker moduli space of canonical models of surfaces with $K^2=2p_g-3$ and intersects

all the components of such closure, and (3) the KSBA moduli space is smooth at a general point of $\mathfrak D$. Moreover $\mathfrak D$ has 1 or 2 irreducible components, depending on the residue class of p_g modulo 4. This is joint work with Rita Pardini.

Combinatorially minimal mori dream surfaces of general type

Jong Hae KEUM (CMC-Center for Mathematical Challenges – KIAS-Korea Institute for Advanced Study)

This talk introduces a new approach to study minimal surfaces of general type with p_g = 0 via their Cox rings, especially using the notion of combinatorially minimal Mori dream space introduced by Hausen.
First, general properties of combinatorially minimal Mori dream surfaces will be given.
Second, several examples of Mori dream surfaces of general type with p_g = 0 will be provided, along with explicit computation of their effective cones.
Then their fibrations and combinatorially minimal models will be discussed.
Many minimal surfaces of general type with p_g = 0 arise from the minimal resolutions of combinatorially minimal Mori dream surfaces.
A joint work with Kyoung-Seog Lee.

Generalized sheaf counting

Young-Hoon KIEM (KIAS - Korea Institute for Advanced Study)

Counting bundles or sheaves with desired properties is a fundamental problem in algebraic geometry. Difficulties arise due to automorphism groups even when the moduli stacks are smooth as in the case of semistable vector bundles over smooth projective curves. In this talk, I will discuss several ways to make sense of intersection pairings on Artin stacks with good moduli spaces. These will lead us to generalized Donaldson invariants and generalized Donaldson-Thomas invariants. Comparison of generalized sheaf counting invariants will be discussed in detail for the singular moduli spaces of vector bundles over curves.

Asymptotic bounds for Ramsey numbers

Jeong Han KIM (KIAS-Korea Institute for Advanced Study - Vice President of KAST-The Korean Academy of Science and Technology)

Ramsey numbers are a cornerstone of combinatorial mathematics, representing the minimum size of a structure required to guarantee a certain property. Specifically, the Ramsey number, denoted as R(s,t), is the smallest integer such that any red-blue edge-coloring of the complete graph contains a monochromatic red or a monochromatic blue subgraphs. Despite their simplicity in definition, Ramsey numbers are notoriously difficult to compute or even estimate, leading to a wealth of research on asymptotic bounds. This survey reviews state-of-the-art results, key techniques, and open problems related to asymptotic bounds for Ramsey numbers. We focus particularly on R(3,t) and R(4,t), including significant contributions of Sam Mattheus and Jacques Verstraete on R(4,t).

General polarized HK varieties of Type $K3^{[a^2+1]}$ as moduli spaces of sheaves

Kieran Gregory O'GRADY (Linceo, Sapienza Università di Roma)

Moduli spaces of sheaves on polarized K3 surfaces (or, when they exist, their crepant desingularizations) provide examples of polarized HK varieties of Type ${\rm K3}^{[n]}$ for arbitrary n (respectively of Type OG10). Since for $n\ge 2$ we have $b_2({\rm K3}^{[n]})=b_2({\rm K3})+1$, and $b_2({\rm OG10})=b_2({\rm K3})+2$, a general polarized HK variety of Type ${\rm K3}^{[n]}$ for $n\ge 2$, or a general polarized HK variety of Type OG10, is \emph{not} a moduli space of sheaves on a polarized K3 surface. We expect that a general polarized HK variety of Type ${\rm K3}^{[a^2+1]}$, for all positive natural numbers a (with some restrictions on its discrete invariants) is a moduli space of sheaves on a polarized HK variety of Type ${\rm K3}^{[2]}$.
We will give a precise statement and outline the main steps of the (expected) proof.

From determinants to Cayley Hamilton algebras

Claudio PROCESI (Linceo, Sapienza Università di Roma)

The theory of Cayley Hamilton algebras is an abstract treatment of the Cayley Hamilton Theorem for matrices. it gives a partial solution to a problem of Malcev to characterize rings which can be embedded in matrix algebras over commutative rings.
I will discuss a recent result and some general questions

Fake homology and homotopy quadrics, Surfaces isogenous to a Product, and numerically trivial automorphisms of surfaces

Fabrizio CATANESE (Linceo, Universität Bayreuth)

Hirzebruch asked about the existence of surfaces homeomorphic to F_0 = P^1 x P^1 which are not rational (hence of general type).
We define fake homotopy quadrics as simply connected minimal surfaces of general type with K^2=8, q=p_g=0.
I will make some remarks on the question, based on joint discussion with J. Yang.
Fake homology quadrics are the minimal surfaces of general type with K^2=8, p_g=q=0.
It is conjectured that these are exactly the surfaces whose universal covering is the bidisk.
There are 4 types: the surfaces isogenous to a product (free quotient of a product of curves) and the irreducible quotients, and, in each class, those with even, respectively odd intersection form.
I will show that the class of fake odd Q-homology quadrics is non empty, using a family of surfaces isogenous to a product.
Fake quadrics isogenous to a product were classified in my joint work with Bauer and Grunewald (with a missing case added by Frapporti) and I will then speak of joint work in progress with Frapporti, showing that for the unmixed type the order |Aut_Q| of the group of automorphisms acting trivially on Q-cohomology is at most 192, and equality is attained exactly for G = (Z/2)^3.
Time permitting, I will then briefly describe joint results with Wenfei Liu and Matthias Schuett, concerning properly elliptic surfaces,
regarding bounds for |Aut_Z |, while showing that there is no absolute bound for |Aut_Q | if p_g=q=0, and we have examples attaining the maximum in the inequality
|Aut_Q | \leq 4s \leq 4 (P_2 + 1) (s is the number of multiple fibres). These contradict earlier results of other authors.

Smoothing of projective schemes with lci singularities

Barbara FANTECHI (Lincea, SISSA Trieste)

This is joint work with Rosa Maria Miró Roig arXiv:2508.04343.
Let X be a projective scheme, and assume that X has lci singularities, which may be non isolated. We give sufficient conditions to ensure that X is (geometrically) smoothable, in particular providing a new and shorter proof to the geometric version by Nobile of Tziolas' formal smoothability criterion.
This follows from showing that the general fiber of a flat proper family with smooth basis and X as special fiber is smooth, under a simple assumption on T^1_X; this is true for proper schemes and DM stacks.
As a corollary we get criteria for smoothability of singular K3 surfaces as well as stable surfaces of general type.

Hyperbolicity of log-surfaces and hypertangency of curves

Lucia CAPORASO (Università di Roma Tre)

Some well-known conjectures of Green-Griffiths, Kobayashi, Lang-Vojta (and others) state that a surface S of general type should contain finitely many (in some cases none at all) rational curves. Such conjectures remain interesting (and open) when extended to a log-surface (S,B) of log-general type.
We shall discuss the case where B is a reducible divisor and report on recent progress, including some joint work with Amos Turchet.

Asymptotic directions in the moduli space of curves

Paola FREDIANI (Università di Pavia)

I will report on a joint work with E. Colombo and G.P. Pirola, where we study asymptotic directions in the tangent bundle of the moduli space of curves of genus g, namely those tangent directions that are annihilated by the second fundamental form of the Torelli map. I will exhibit examples of asymptotic directions for any g at least 4. I will show that if the rank d of a tangent direction at [C] (with respect to the infinitesimal deformation map) is less than the Clifford index of the curve C, then the tangent direction is not asymptotic. Finally I will determine all asymptotic directions of rank 1 and give an almost complete description of asymptotic directions of rank 2.

Surfaces with canonical map of odd degree

Rita PARDINI (Università di Pisa)

I will report on joint work in progress with M. Mendes Lopes and R. Pignatelli.
Let S be a smooth minimal complex surface of general type. Assume that the canonical map is generically finite of degree d onto a surface Y. By classical results of Xiao and Beauville, it is known that $d < 9$ if the geometric genus of S is large enough, and for $d = 8$ there are examples with unbounded invariants, showing that the bound is sharp.
Families of examples with $d = 2, 4, 6$ and unbounded geometric genus are also known, while there are only sporadic examples with d odd.
I will report on work in progress aiming at explaining this asymmetry.
Under the additional assumption that the general canonical curve of S is smooth, we are able to: (1) give an explicit upper bound on on the geometric genus in case $d = 7$; (2) in

the case when the canonical image Y is ruled by lines, show that the geometric genus is at most d + 2 and Y is a cone over the rational normal curve of degree.
For d = 3 surfaces as in (2), i.e., with Y ruled by lines, were completely described by Mendes Lopes and Pardini and, independently, by Starnone: there are two main examples, with geometric genus 5, respectively 4, and examples with lower invariants are obtained by degenerating these two types of surfaces. For d = 5 and Y ruled by lines a similar picture emerges from a theoretical analysis of thesituation. However, quite surprisingly, we are able to rule out the existence of most of the possibilities and we have not yet been able to produce examples.

Unipotent group actions on affine hypersurfaces

Jihun PARK (Pohang University of Science and Technology - POSTECH)

As an additive analogue of toric geometry, unipotent group actions --particularly additive actions--on varieties are very attractive objects of study. Indeed, additive-actions have been investigated not only for their intrinsic interest, but also for their connections to the theory of rational singularities. For example, it is known that a Brieskorn--Pham surface singularity is a cyclic quotient singularity if and only if the surface admits a nontrivial regular additive action. Considering the three-dimensional analogue, Flenner and Zaidenberg posed in 2003 the following question:
Does the affine Fermat cubic threefold $x^3+y^3+z^3+w^3=0$ in $\mathbb{A}^4$ admit a nontrivial regular additive action?
Although deceptively simple, this problem remained open for ten years. In 2013, following partial results obtained by Kishimoto, Prokhorov, and Zaidenberg, together with Cheltsov and Won, I gave a negative solution. We proved that the affine cone over any smooth hypersurface in the following three families does not admit a nontrivial additive action:
1) smooth cubic hypersurfaces in P(1,1,1,1),
2) smooth quartic hypersurfaces in P(1,1,1,2),
3) smooth sextic hypersurfaces in P(1,1,2,3).
In this talk, we present an extension of the 2013 result to the setting of quasismooth del Pezzo hypersurfaces in weighted projective 3-spaces.
In particular, we prove that any quasismooth del Pezzo hypersurface of index 1 does not contain an anticanonical cylinder.
This work is in collaboration with Adrien Dubouloz.

A study of rational homology projective planes

Jongil PARK (Seoul National University - SNU)

A normal projective surface with the same Betti numbers of the projective plane CP2 is called a rational homology projective plane (briefly Q-homology CP2 or QHCP2). People working in algebraic geometry and topology have long studied a Q-homology CP2 with possibly quotient singularities. It has been known that it has at most five such singular points, but it is still mysterious so that there are many unsolved problems left. In this talk, I'll review some known results and open problems in this field which might be solved and might not be solved in near future. In particular, Id like to review the following two topics and to report some recent progress:
1. Algebraic Montgomery-Yang problem.
2. Classification of Q-homology CP2 with quotient singularities.
This is a joint work with Woohyeok Jo and Kyungbae Park.